$f(x)$ is continuous at $x=a$ if

$$\lim_{x\to a} f(x) = f(a)$$

Def: f is continuous on an interval I if it is continuous at every number a in the interval I.

Ex: $f(x) = \dfrac{2x^2-5x-3}{x-3}$

① Where is f discontinuous?

For rational fcns this is when the denominator is zero.

$$x-3=0 \longrightarrow x=3.$$

② On what interval(s) is f continuous?

$$(-\infty, 3) \cup (3, \infty)$$

③ $\lim_{x \to 5} \frac{2x^2 - 5x - 3}{x - 3} = \frac{2 \cdot 25 - 25 - 3}{5 - 3}$

$= \frac{22}{2} = 11$

Ex: $f(x) = \frac{\sqrt{x^2 - 4} + x^2}{\sqrt{x + 7}}$

Where is f ~~dis~~continuous?

$\sqrt{x^2 - 4}$

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$|x| \geq 2$

$x \geq 2$ $\quad$ $-x \geq 2$

$\downarrow$

$x \leq -2$

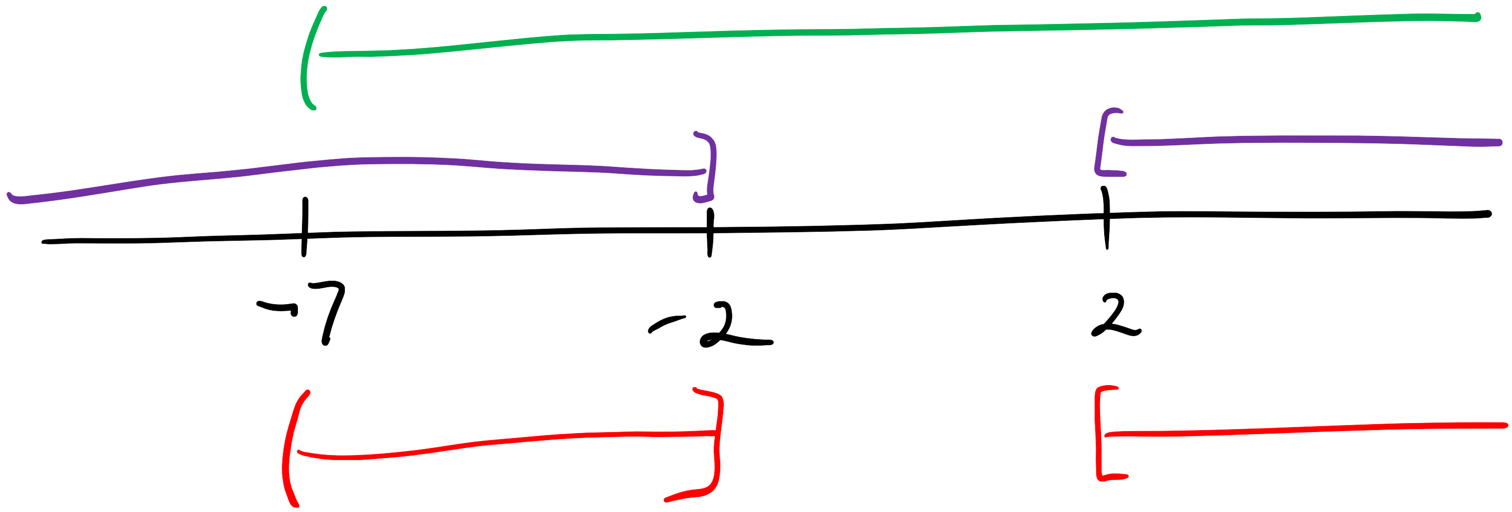
$*\sqrt{x^2} = |x|$

$\sqrt{x+7}$: in the denom.

$\downarrow$

$x + 7 > 0$

$x > -7$



f is continuous on: $(-7,-2] \cup [2,\infty)$

Theorem

If f is continuous at $x=b$ and $\lim_{x\to a} g(x) = b$, then

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$$

Corollary

If g is continuous at a and f is continuous at $g(a)$, then $f\circ g$ is continuous at a.

Ex: $\underbrace{\lim_{x\to 1} \ln\left(\frac{5-x^2}{1+x}\right)}$ $= \ln(2)$

$$\underbrace{\lim_{x\to 1} \frac{5-x^2}{1+x}} = \frac{5-1^2}{1+1} = 2$$

Is $\ln(x)$ continuous at $x=2$? Yes

(Domain of $\ln(x)$ is $(0,\infty)$)

So $\lim_{x\to 1} \ln\left(\frac{5-x^2}{1+x}\right) = \ln\left(\lim_{x\to 1} \frac{5-x^2}{1+x}\right)$

$= \boxed{\ln(2)}$

Domains of common functions

Any polynomial $\longrightarrow (-\infty, \infty)$

sin, cos $\longrightarrow (-\infty, \infty)$

e^x, b^x $\longrightarrow (-\infty, \infty)$

$(b>0, b\neq 1)$

arctan $\longrightarrow$ $(-\infty, \infty)$

rational fcns $\longrightarrow$ when denominator $\neq 0$

tan, sec $\longrightarrow$ when $\cos \neq 0$

cot, csc $\longrightarrow$ when $\sin \neq 0$

arcsin $\longrightarrow$ $[-1, 1]$

ln, $\log_b$ $\longrightarrow$ $(0, \infty)$
$(b > 0, b \neq 1)$

$x^{1/2}, x^{1/4}, x^{1/6} \longrightarrow [0, \infty)$

$x^{1/3}, x^{1/5}, x^{1/7} \longrightarrow (-\infty, \infty)$